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On formation of a magnetic field by vortical processes in stellar plasma

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1 Introduction

According to the concepts of present-day cosmology, stars and magnetic fields in them are believed to originate and form at the same time. The mechanism of origin of magnetic fields in a primordial gaseous-dust cloud and an originating star is open to question. Further, it seems unquestionable that magnetic fields are interrelated with currents in the stellar plasma. This is why, the effect of friction caused by resistance must result in the diminishing of the currents and thereby of magnetic induction in long-duration existence of a star. A number of "dynamo" mechanisms that lead to strengthening of a magnetic field have been substantiated theoretically. By some notions, the presence of such mechanisms in a star may compensate for friction losses (Priest 1985). However, it is doubtful that these mechanisms do actually occur widely and are efficient in stars.

Besides, all these "dynamo" mechanisms have one common disadvantage: they are not operative if the initial magnetic field is absent altogether. For this reason, it has to be assumed that the energy of the magnetic field of a star does not drop below a certain threshold value. If this does occur, then the magnetic field cannot be restored even at powerful dynamic processes in the star. This appears to be improbable and is likely to be inconsistent with observational data, in particular, for the Sun. Thus, the problem of origin and keeping of the magnetic field in stars proves to be quite urgent and is far not only from the solution, but also from the universally adopted interpretation.

The present paper advances for discussion a mechanism of origin of a magnetic field in the stellar plasma from a "zero" state. The currents caused by vortical processes in the stellar plasma are shown to be accompanied by the origin of a related magnetic field.

2 General characteristics of stellar plasma

To simplify the posed problem, we consider that matter in the region studied, where the vortex process takes place, is constituted from ionized hydrogen. Within the framework of the problem being discussed, this assumption holds sufficiently true for the Sun, in which, as far as the number of atoms is concerned, hydrogen accounts for more than 87%. Hydrogen is mostly ionized at the temperature $T > 100000\,\mathrm{K}$ (Priest 1985; Lifshits, Pitaevsky 1979).

The hydrogen plasma is a combination of two components: monoatomic gas of protons and monoatomic gas of electrons. Designate the concentration (specific number per unit volume) of electrons as n_e, and protons as n_i. The specific volume charge of electrons is $q_e = -n_e e$, while that of protons is $q_i = n_i e$ ($e = 1.602 \cdot 10^{-19}$C is the elementary charge). The net specific charge has the density $q = q_i - q_e$.

The potential φ and the intensity $\bar{E}_n = -grad\,\varphi$ of the electric field in the plasma depend on the density of the net charge ($\bar{E}_n = div\,q/\varepsilon_0$, $\varepsilon_0 = 8.85 \cdot 10^{-12}\,\mathrm{F/m}$ is the electric constant) (Matveev 1980). Having originated, the potential electric field prevents from further division of charges. Because of the large elementary charge, the difference in concentrations of protons and electrons is small $\mid n_i - n_e \mid << n_i$ (Priest 1985). In most of the relationships this difference is insignificant. This is why, unless otherwise specified, we consider the concentrations of protons and electrons to be equal, $n \equiv n_i \cong n_e$.

The electric forces affecting unit volumes of protons and electrons are

$$\bar{f}_i = -ne\ grad\,\varphi, \qquad \bar{f}_e = ne\ grad\,\varphi = -\bar{f}_i, \tag{1}$$

respectively.

The mass density of the electron gas is $\rho_e = m_e n$, ($m_e = 9.1091 \cdot 10^{-31}$ kg is the mass of the electron), while the mass density of the proton gas is $\rho_i = m_i n$ ($m_i = 1.67252 \cdot 10^{-27}$ kg is the mass of the proton). The ratio of densities of the gases corresponds to that of the masses of the electron and proton

$$\frac{\rho_e}{\rho_i} = \frac{m_e}{m_i} = \frac{1}{1836}. \tag{2}$$

The pressures of the gases of protons and electrons are equal to $p_i = n_i k T_i$, $p_e = n_e k T_e$, respectively ($k = 1.3805 \cdot 10^{-23}$ J/K is Boltzmann's constant, T_i, T_e are the temperatures of the gases) (Lifshits, Pitaevsky 1979). These gases occupy one and the same volume, rapid heat exchange occurs between them. This is why the proton and electron gases are at thermal equilibrium, and their temperatures are practically equal everywhere, $T_i = T_e \equiv T$, although they may change over the volume of the star. Thus, the equal partial pressures of the proton and electron gases (since $n_i = n_e$), following Dalton's law are equal to half the total pressure of the plasma, $p_i = p_e = p/2$.

In the field of gravity the specific force $\overline{f}_e = \rho_e \overline{g}$ has an effect on the electron gas, while the proton gas is affected by the force $\overline{f}_i = \rho_i \overline{g}$, where $\overline{g} = -g\overline{r}/r$ is the acceleration of gravity directed towards the center. As is known (Lojtsyansky 1987), the gravity acceleration has a potential function $\overline{g} = -grad\,G$. In a particular case, where the acceleration is constant, $G = gr$.

3 Viscosity of hydrogen plasma and its components

The viscosity of the components of the hydrogen plasma is characterized by the dynamic viscosity η_i of the proton gas and η_e of the electron gas and also by the coefficient of mutual friction of the gases η. The mutual motion of the gases of protons and electrons occurs under the action of forces equal in value but of opposite directions. We believe that the force $\bar{\tau}_i = d\bar{F}/dV$ has an effect on a unit volume of the proton gas, while the electron gas is under the action of the force $\bar{\tau}_e = -\bar{\tau}_i$. The external force that influences the gas of protons is counterbalanced by the force of friction acting upon this gas from the gas of electrons, $\bar{\tau}_{ei} = -\bar{\tau}_i$. The same holds for the gas of electrons, $\bar{\tau}_{ie} = -\bar{\tau}_e$.

The difference $\bar{v}_i - \bar{v}_e$ between the mean velocities of protons, $\bar{v}_i$, and electrons, $\bar{v}_e$, depends on the indicated specific forces and the coefficient of mutual friction $\eta (N \cdot s/m^4)$

$$\bar{\tau}_i = -\bar{\tau}_e = \eta \left(\bar{v}_i - \bar{v}_e\right). \tag{3}$$

The motion of protons and electrons gives rise to corresponding currents with densities $\overline{j}_i = ne\bar{v}_i$ and $\overline{j}_e = -ne\bar{v}_e$. The total current density in the stellar plasma depends on the sum of these densities

$$\overline{j} = \overline{j}_i + \overline{j}_e = ne \left(\bar{v}_i - \bar{v}_e\right).$$

With this equality taken into account, formula (3) may be rewritten as follows:

$$\bar{\tau}_i = -\bar{\tau}_e = \frac{\eta}{ne}\overline{j}. \tag{4}$$

The value of the mutual viscosity coefficient η and the plasma conductivity are interrelated. To prove this, note that at some arbitrary intensity $\overline{E}$ the electric field affects the volume electric charge of the proton gas with a specific force $\bar{\tau}_i = \overline{E}\,ne$. Under the action of these forces (if other forces of non-electric origin are absent) the mutual motion of electrons and protons occurs in accordance with (Lojtsyansky 1987)

$$\overline{E}\,ne = \eta \left(\bar{v}_i - \bar{v}_e\right).$$

Multiply both sides of this equality by ne

$$\overline{E}\,n^2 e^2 = \eta\, n\, e\, \left(\bar{v}_i - \bar{v}_e\right).$$

By extracting here the cofactors corresponding to the current density, obtain

$$\overline{j} = \frac{n^2 e^2}{\eta}\overline{E}. \tag{5}$$

Ohm's law is written in a differential form in the following manner $\overline{j} = \gamma \overline{E}$ (γ is the conductivity). Consequently, the coefficient at the intensity in (5) is the plasma conductivity and

$$\gamma = n^2 e^2/\eta, \quad \eta = n^2 e^2/\gamma. \tag{6}$$

The viscosities of the proton and electron gases increase sharply with temperature (as $T^{5/2}$) (Lifshits, Pitaevsky 1979). However, the ratio of viscosities remains unchanged and is defined by the square root of the mass ratio of the proton and electron (Lifshits, Pitaevsky 1979):

$$\frac{\eta_e}{\eta_i} \equiv \sqrt{\frac{m_e}{m_i}} = \frac{1}{42.8}. \tag{7}$$

The coefficient of mutual friction η decreases with increasing temperature. However, it is very large as compared to the viscosity coefficients of the proton and electron gases in the temperature range $10^5 - 10^7\,K$. That is why, the value of the velocity difference between the gases of protons and electrons is smaller than the plasma resultant velocity, determined by the velocity of the proton gas, $\bar{v} \approx \bar{v}_i$

$$|\bar{v}_i - \bar{v}_e| < |\bar{v}|\,. \tag{8}$$

In (Priest 1985) is presented a relationship between plasma conductivity and temperature

$$\gamma = 1.53 \cdot 10^{-2} \frac{T^{3/2}}{L_e}, \tag{9}$$

in the ranges of variations of temperatures $10^4 - 10^7$ K and concentrations of particles $10^{12} - 10^{27}\ m^{-3}$ Coulomb multiplier has been calculated as $5.54 < L_e < 25.1$.

4 Hydrodynamics of plasma at a weak magnetic field

Within the framework of the primary treatment we consider the plasma conductivity to be low and the originating magnetic field to be weak. Then, the forces arising from the interaction between the currents in the plasma and the magnetic field are comparatively weak, and the inductive currents are practically absent. If the forces resulting from the interaction between the magnetic field and the currents were not taken into account, then the laminar flow of incompressible plasma could be described by modified Navier–Stokes equations (Lojtsyansky 1987)

$$\rho_i \frac{d\bar{v}_i}{dt} = -grad\,(p/2) - ne\,grad\,\varphi - \rho_i\,grad\,G + \eta_i \Delta \bar{v}_i + \bar{\tau}_{ei} \tag{10}$$

$$\rho_e \frac{d\bar{v}_e}{dt} = -grad\,(p/2) + ne\,grad\,\varphi - \rho_e\,grad\,G + \eta_e \Delta \bar{v}_e + \bar{\tau}_{ie}. \tag{11}$$

The former was written for the proton gas, the latter for the electron gas. The equations are related to each other since the addend $\bar{\tau}_{ei}$ defines the specific force of friction acting on the proton gas from the electron gas, while $\bar{\tau}_{ie} = -\bar{\tau}_{ei}$ defines the corresponding counterforce for these gases. These specific forces depend on the difference in the mean velocities of protons and electrons and on the coefficient of mutual friction of the gases η (3).

A set of equations similar to (10, 11) is written in (Kulikovsky, Lubimov 1962) for the general case (the plasma is not completely ionized and not purely hydrogen, the action of the magnetic field is taken into account).

We use the estimates made and the relations for the velocities of the proton and electron gases, pressure, potential electric field force and mutual friction force for substituting into the initial equations (10, 11) :

$$\rho_i \frac{d\bar{v}}{dt} = -grad(p/2) - ne\,grad\ \varphi - \rho_i\,grad\,G + \eta_i \Delta \overline{v} - \frac{ne}{\gamma}\bar{j}, \tag{12}$$

$$\rho_e \frac{d\bar{v}}{dt} = -grad\,(p/2) + ne\,grad\ \varphi - \rho_e\,grad\,G + \eta_e \Delta \overline{v} + \frac{ne}{\gamma}\bar{j}. \tag{13}$$

The equations of continuity for the charge and mass density have, respectively, the form (Matveev 1980; Lifshits, Pitaevsky 1979):

$$\frac{\partial q}{\partial t} + div\,\bar{j} = 0, \quad \frac{\partial \rho}{\partial t} + div\ (\rho\,\overline{v}) = 0. \tag{14}$$

Since in stationary and close to stationary dynamic processes the local mass density is not time variable ($\partial\rho/\partial t = 0$), and the charge density in all processes is close to zero ($q \approx 0$), then the set of equations (10, 11) should be added by the equalities

$$div\,\overline{j} = 0, \quad div\,\overline{v} = 0. \tag{15}$$

Summing up equations (13, 14), and with allowance made for the previous estimates, obtain

$$\rho\frac{d\bar{v}}{dt} = -grad\,p + \rho\overline{g} + (\eta_i + \eta_e)\,\Delta\bar{v}, \tag{16}$$

where $\overline{v} = \overline{v}_i \approx \overline{v}_e$, $\rho \approx \rho_i$ are the velocity and density of the plasma, respectively.

In the absence of a magnetic field in the completely ionized hydrogen plasma electric properties of protons and electrons cancel out and prove to be insignificant with respect to hydrodynamic parameters, and the influence of the electron gas, as compared to the gas of protons, causes the pressure to rise ($p_i \to p = 2p_i$) and results in some increase in the friction coefficient.

To exclude the Laplacian of velocity, multiply both sides of equation (12) by the multiplier

$$\frac{\eta_e}{\eta_i} = \frac{1}{42.8}$$

and then subtract equation (13) from the obtained equality. Derive the following relationship:

$$\sqrt{\rho_i\rho_e}\frac{d\bar{v}}{dt} = (1 - \frac{\eta_e}{\eta_i})\ grad\,(p/2) - (1 + \frac{\eta_e}{\eta_i})\ \cdot ne\,grad\,\varphi\ - \sqrt{\rho_i\rho_e}\,grad\,G - (1 + \frac{\eta_e}{\eta_i})\frac{ne}{\gamma}\overline{j}, \tag{17}$$

where it is taken into account that the density ratio of the proton and electron gases (2), equal to 1/1836, is essentially smaller than the value of the indicated multiplier.

Since the quantity

$$\frac{\eta_e}{\eta_i} \cong \sqrt{\frac{\rho_e}{\rho_i}} = \frac{1}{42.8}$$

is small in comparison with unity, it may be neglected in the multipliers. Then obtain

$$\sqrt{\rho_i\rho_e}\frac{d\bar{v}}{dt} = grad\,(p/2) - ne\,grad\varphi - \sqrt{\rho_i\rho_e}grad\,G - \frac{ne}{\gamma}\overline{j}. \tag{18}$$

Perform the operation *rot* on both sides of this equality. Here we consider that the densities of the electron and proton gases, plasma conductivity and concentration of particles change with distance slowly and are actually constant coefficients in the differential functions grad in (18). Since $rot\,grad\psi = 0$, we have

$$\sqrt{\rho_i\rho_e}\frac{d\bar{\Omega}}{dt} = -\frac{ne}{\gamma}rot\overline{j}, \tag{19}$$

where $\overline{\Omega} = rot\overline{v}$ is the vorticity of the plasma.

Substituting the equalities $\rho_i = nm_i,\ \ \rho_e = nm_e$ into (19), reduce this formula to the following form

$$rot\ \overline{j} = -a\gamma\frac{d\bar{\Omega}}{dt}, \tag{20}$$

where $a = \sqrt{m_i m_e}/e = 2.436 \cdot 10^{-10}$ kg/C.

The first equation of Maxwell for statics has the form $rot\bar{B} = \mu_0\overline{j}$, ($\overline{B}$ is the magnetic induction, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant) (Matveev 1980). Substituting the current density from this equality into (20), obtain

$$rot\,rot\,\bar{B} = -a\mu_0\gamma\frac{d\bar{\Omega}}{dt}. \tag{21}$$

Using the known formula

$$grad\,div\,\overline{B} = rot\,rot\,\overline{B} + \Delta\overline{B}$$

and taking into account that $div\,\bar{B} = 0$ (Lifshits, Pitaevsky 1979), obtain

$$\Delta \bar{B} = a\mu_0 \gamma \frac{d\bar{\Omega}}{dt}. \tag{22}$$

The set of Navier–Stokes equations (10, 11) is reduced to two separate equations: the known (Priest 1985; Kulikovsky 1962) Navier–Stokes equation for the plasma as a whole (16) and the one (22) that we obtained for the first time, which characterizes the relation between vorticity and induction of the field initiated by the vortical process. The magnetic induction is in direct proportion with the plasma conductivity. That is why, the original condition that the magnitude of induction of a field being formed should be small is complied with at low plasma conductivity.

The difference between the viscous (7) and inertial (2) properties of the proton and electron gases formed a foundation for deriving (22). Turbulent viscosities of the proton and electron gases may be by several orders of magnitude higher than molar. However, as it follows from the theory of the way of mixture of Prandtl (Lojtsyansky 1987), the ratio of turbulent viscosities is the same as that of molecular viscosities. For this reason, the final relationship (22) characterizes the connection between plasma vorticity and induction of a magnetic field being formed not only in laminar, but also in turbulent motion.

5 Complete set of equations of magnetohydrodynamics in vortical processes

As is known (Matveev 1980), the induced electromotive force (e.m.f.) appears as a conductor moves in a magnetic field. This e.m.f. is characterized by the intensity $\overline{E} = [\overline{v}\overline{B}]$ equal to the vector product of the conductor speed $\overline{v}$ into magnetic field induction $\overline{B}$.

Apply the indicated formula separately to the motion of the proton and electron gases. Derive that the corresponding intensities are $\overline{E}_i = [\overline{v}_i\overline{B}]$, $\overline{E}_e = [\overline{v}_e\overline{B}]$. The specific volume forces affecting the gases because of interaction with the magnetic field are $\overline{f}_i = ne\,[\overline{v}_i\overline{B}]$, $\overline{f}_e = -ne\,[\overline{v}_e\overline{B}]$. Introducing the corresponding summands into equations (12, 13), obtain the initial set of equations of magnetohydrodynamics of the fully ionized hydrogen plasma

$$\rho_i \frac{d\bar{v}}{dt} = -grad(p/2) - ne\, grad\,\varphi - \rho_i\, grad\, G + \eta_i \Delta\bar{v} - \frac{ne}{\gamma}\bar{j} + ne\,[\overline{v}_i\overline{B}], \tag{23}$$

$$\rho_e \frac{d\bar{v}}{dt} = -grad\,(p/2) + ne\, grad\;\varphi - \rho_e\, grad\, G + \eta_e \Delta\bar{v} + \frac{ne}{\gamma}\bar{j} - ne\,[\overline{v}_e\overline{B}]. \tag{24}$$

Summing up these equations, derive an equation of magnetohydrodynamics of the plasma as a whole with allowance for gravity, viscosity and forces of interaction of the magnetic field with the currents (Priest 1985; Kulikovsky, Lubimov 1962)

$$\rho \frac{d\bar{v}}{dt} = -grad\, p + \rho\overline{g} + (\eta_i + \eta_e)\,\Delta\bar{v} + [\bar{j}\,\overline{B}]. \tag{25}$$

When adding together the last summands of equations (24, 25), the following chain of equalities was taken into account

$$ne\,[\overline{v}_i\,\overline{B}] - ne\,[\overline{v}_e\,\overline{B}] = [(ne\,\overline{v}_i)\,\overline{B}] + [(-ne\,\overline{v}_e)\,\overline{B}] = [\bar{j}_i\,\overline{B}] + [\,\bar{j}_e\,\overline{B}] = [(\bar{j}_i + \bar{j}_e)\,\overline{B}] = [\bar{j}\;\overline{B}].$$

As in deriving equation (18), multiply (23) into the multiplier

$$\frac{\eta_e}{\eta_i} \equiv \sqrt{\frac{m_e}{m_i}} = \frac{1}{42.8}$$

and after that subtract equation (24) from the equality obtained. Derive the following actually exact equation:

$$\sqrt{\rho_i\rho_e}\frac{d\bar{v}}{dt} = grad\,(p/2) - ne\, grad\;\varphi - \sqrt{\rho_i\rho_e} grad\, G - \frac{ne}{\gamma}\bar{j} + ne\,[\overline{v}\,\overline{B}]. \tag{26}$$

The following chain of equalities was allowed for:

$$\frac{1}{42.8} ne\,[\overline{v}_i\,\overline{B}] + ne\,[\overline{v}_e\,\overline{B}] \approx ne\,[\overline{v}_e\,\overline{B}] \approx ne\,[\overline{v}\;\overline{B}].$$

Take the operation *rot* from both sides of (26):

$$\sqrt{\rho_i \rho_e}\frac{d\bar{\Omega}}{dt} = -\frac{ne}{\gamma} rot\ \bar{j} + ne\, rot\, [\overline{v}\, \overline{B}]. \tag{27}$$

Having executed transformations similar to the transition from (19) to (22), we have

$$rot\bar{j} = -a\gamma \frac{d\bar{\Omega}}{dt} + \gamma\, rot\, [\overline{v}\, \overline{B}]. \tag{28}$$

By performing further transformations, derive

$$\Delta \bar{B} = a\mu_0 \gamma \frac{d\bar{\Omega}}{dt} - \mu_0 \gamma\, rot\ [\overline{v}\, \overline{B}]. \tag{29}$$

Thus, the complete magnetohydrodynamics model of a vortical process is described by the known equation (25) and equation (29) that we have derived. Equation (29) is distinguished from partial equation of hydrodynamics (22) by the presence of a summand taking account of the back action of the magnetic field on the process of its changing. As it can be readily shown, at low plasma conductivity γ the simple model (22) is sufficient, while at high conductivity the general model (29) should be employed.

6 The reality of the model proposed

Considering in advance a magnetic field being formed to be weak, estimate the maximum magnitude of induction using the hydrodynamic model (22). Calculate the magnetic induction magnitude of the originating magnetic field in an axially symmetric flat stationary process of the type of vortex-flow (Fig. 1) or vortex-spring (Fig. 2). For this process the components $\overline{B}, \overline{\Omega}$ in the plane x, y are identically equal to zero. Therefore, equation (22) holds only for z components

$$\Delta B_z = a\mu_0 \gamma \frac{d\Omega_z}{dt}. \tag{30}$$

Taking into account that $B_y = B_z = 0$, $\Omega_x = \Omega_y = 0$, representing the Laplacian of induction B_z and the full derivative of vorticity Ω_z in the cylindrical coordinate system (Kulikovsky, Lubimov 1962) and omitting index z, come to the following expression

$$\frac{1}{r}\frac{\partial\left(r\frac{\partial B}{\partial r}\right)}{\partial r} = a\mu_0\gamma\left(\frac{\partial \Omega}{\partial t} + \frac{\partial \Omega}{\partial r} v_r\right). \tag{31}$$

Here r is the radius, v_r is the radial component of the gas velocity. In consequence of stationarity $\partial\Omega/\partial t = 0$. For an axially symmetric flow the z component of vorticity Ω is equal (Lojtsyansky 1987) to

$$\Omega = \frac{\partial v_t}{\partial r} + \frac{v_t}{r},$$

where v_t is the tangential (peripheral) component of the gas velocity. In solid-body rotation the vorticity is unchanged everywhere, $\Omega(r) = \partial v_t/\partial r = v_t/r = const$, while the tangential velocity is directly proportional to the distance from the center. In vortex-free rotation the vorticity is equal to zero everywhere $\partial v_t/\partial r = -v_t/r$, $\Omega(r) = 0$, while the tangential velocity of the gas is inversely proportional to the distance. These two extreme causes correspond to a very high gas viscosity (then $v_r = 0$) or no viscosity. They are not complied with in the real stellar gas. To make estimates, we will assume that in some intermediate case the tangential velocity is independent of the radius, $v_t(r) = const$. Then

$$\Omega = \frac{v_t}{r}.$$

In accordance with the continuity condition (Lojtsyansky 1987) the power of the source, $Q = 2\pi r v_r(r) = const$, is unchanged (at vortex-flow $Q < 0$), and the radial velocity is, therefore, inversely proportional to the radius. With allowance made for the presented relations, equality (30) changes to

$$\frac{\partial\left(r\partial B/\partial r\right)}{\partial r} = a\mu_0\gamma\frac{Q}{2\pi}\left(\frac{\partial\left(v_t/r\right)}{\partial r}\right). \tag{32}$$

Excluding left and right differentiation with respect to r, derive a relationship between the induction derivative and the radius

$$\frac{\partial B}{\partial r} = a\mu_0\gamma\frac{Q}{2\pi}v_t\frac{1}{r^2}. \quad (33)$$

Considering the induction at an infinite distance to be zero, take the integral over (33) to a certain value of the radius R

$$B\,(R) = a\mu_0\gamma v_t\frac{Q}{2\pi}\int\limits_{\infty}^{R}\frac{dr}{r^2} = -a\mu_0\gamma v_t\frac{Q}{2\pi R} = -a\mu_0\gamma v_t v_r(R). \quad (34)$$

As it follows from (34), the magnetic field direction at vortex-flow ($v_r < 0$) is defined by the right-hand-screw (gimlet) rule, proceeding from the direction of the peripheral plasma velocity (Fig. 1). At vortex-spring ($v_r > 0$) the magnetic field direction is determined by the left-hand-screw rule (Fig. 2).

The induction is measured in Tesla

$$|B| = |a|\ |\mu_0|\ |\gamma|\ |v|^2 = \frac{kg\,H\,m^2}{C\,m\,s^2\,\Omega\,m}\frac{kg\,\Omega\ s}{C\ s^2\,\Omega} = \frac{kg}{A\,s^2} = \frac{N}{A\,m} = T.$$

To evaluate the magnetic induction in the central region of the vortical process, specify the value of the radius R_0 at which the peripheral and radial velocities of the stellar gas become equal in magnitude $|v_r(R_0)| = |v_t| = |Q|/(2\pi R_0)$. Then

$$B\,(R_0) = a\mu_0\gamma v_t^2. \quad (35)$$

The case $v_r > v_t$ under real conditions seems to be unlikely. Most likely, when approaching this threshold, the gas flow takes the axial direction, and, therefore the plane process model becomes inapplicable.

The temperature T=100000 K, in the upper layer of the Sun is thought to be the basic parameter. Assuming the Coulomb multiplier to be 10, obtain the value of the plasma conductivity (9)

$$\gamma = 1.53 \cdot 10^{-3}(10^5)^{3/2} = 4.84 \cdot 10^4\ \Omega^{-1}\,m^{-1}\ .$$

For estimating the plasma motion velocity in the vortical, Bernulli's formula is used

$$p + \rho v^2/2 = p_0,$$

where p_0 is the magnitude of the so-called total pressure on the streamline. The gas may be treated as incompressible if the amount of dynamic pressure accounts for a small part of the total. It will be assumed that at a maximum velocity this relation is of the same order as in powerful cyclonic processes on Earth

$$\rho v^2/2 = 0.1p_0.$$

Since $\rho \approx m_i n$, $p_0 = 2nkT_0$, then $v^2 = 0.4kT_0/m_i$. The plasma velocity

$$v = \sqrt{0.4kT_0/m_i} = \sqrt{\frac{0.4 \cdot 1.38 \cdot 10^{-23} \cdot 10^5}{1.67 \cdot 10^{-27}}} = 18\ km/s$$

corresponds to the temperature specified above. Velocities of this and higher order at the surface of the Sun are recorded in real observations (Priest 1985).

Since we adopted $v_r = v_t$, then

$$v_t^2 = v^2/2 = 1.62 \cdot 10^8 m^2/s^2.$$

Substituting numerical values into (31), obtain

$$B = 2.436 \cdot 10^{-10} \cdot 4 \cdot 3.14159 \cdot 10^{-7} \cdot 4.84 \cdot 10^4 \cdot 1.62 \cdot 10^8 = 0.0015\text{T} = 15\text{G}.$$

The value of 15G is quite realistic and the magnetic induction of such an order of magnitude is recorded at the surface of the Sun by direct observations.

Note that the direction of the inductive current (its density is $\overline{j}_m$) is coincident with that of the tangential component of the intensity of the electric field $\overline{E}_{mt}$ due to the flow of plasma in a magnetic field. The radial component $\overline{E}_{mr}$ of the intensity $\overline{E}_m$ caused by the plasma motion is compensated by the potential electric field. The direction of the inductive current $\overline{j}_m$ defined proceeding from inertial properties of the plasma (Fig. 1, 2) Thus, the induction results in intensification of the initial current (but not in its compensation), and thereby, in strengthening of the arising magnetic field. This permits a conclusion to be drawn that the value of magnetic induction computed above is the lower threshold of the maximum under the selected original conditions.

Specific calculation of inductive currents and forces of interaction between the magnetic field and the currents in accordance with the full magnetohydrodynamic model (29) is the problem falling out of the scope of the present paper.

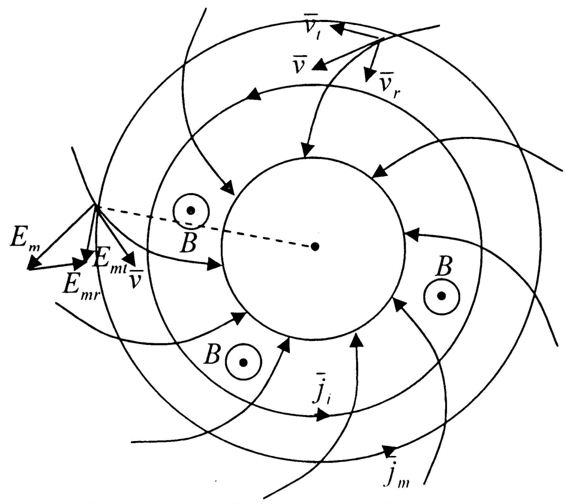

Figure 1: *Magnetic field at vortex flowing.*

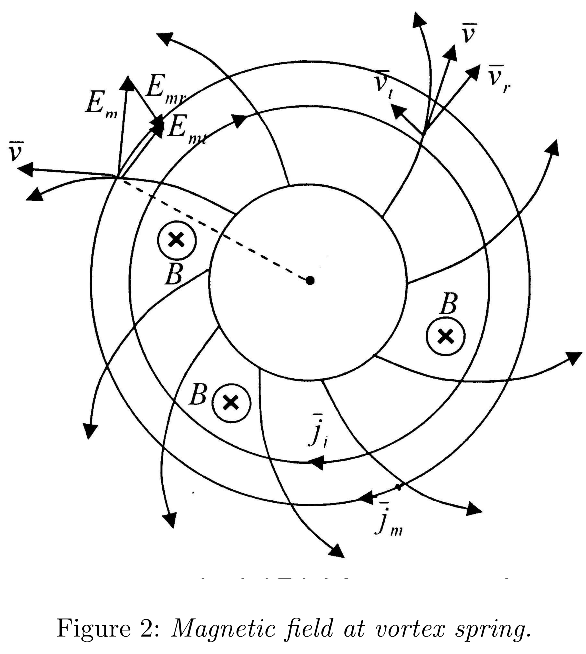

Figure 2: *Magnetic field at vortex spring.*

7 Conclusions

It is shown that in vortex processes in the stellar plasma a magnetic field originates even though it was primarily absent. When testing the verity of the suggested model, an estimate was made of the magnetic induction in a characteristic vortex process in the upper layer of the Sun. The plasma temperature of 100000 K was taken as the initial parameter. The computed magnetic induction (15G) is close that actually observed on the Sun.

The present paper contains the posing of the problem. A number of problems following from this posing need to be discussed in more detail, in particular:

- acquisition and reduction of data of observations and measurements concerning vortex processes and their associated magnetic fields in stars;
- detailed consideration of the role of turbulence in vortex processes;
- construction of a model for partially ionized plasma of complex composition (not entirely hydrogen);
- calculations of magnetic field with allowance for inductive currents at a given structure of vortex flows;
- development of a model and complete calculation of the structure, dynamics and magnetic field arising in vortical processes caused by given thermodynamic conditions under zero and, consequently, non-zero initial and peripheral conditions with taking account of the back action of the magnetic field on vortex flows (the final and most complicated theoretical problem);
- comparison of observational and theoretical data.

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