On some exotic properties of hybrid stars

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Abstract We consider two “wonders” of hybrid stars, i.e. stars which contain a core made of quark matter. First, we explain the existence of a very small region on the mass–radius (M-R) diagram of hybrid stars where all of the lines representing the sequences of models with different values of the bag constant B intersect. This circumstance is shown to be a consequence of the linear dependence of pressure on energy density in the quark cores of hybrid stars. Second, we show that the unusual thermodynamic properties of matter within the region of two-phase coexistence in hybrid stars result in a change of the standard condition for beginning of convection. In particular, the thermal flux transported by convection may be directed towards the stellar center. We discuss favorable circumstances leading to such an effect of “inverse” convection and its possible influence on the thermal evolution of hybrid stars.

Keywords: Hybrid Star, Quark Matter, Convection

1. Introduction

Neutron stars, together with stellar mass black holes, develop from collapsing cores of massive stars at the final stages of their evolution. The birth of a neutron star most likely manifests itself as a supernova explosion. The major part of matter in neutron stars proves to be in an extreme state with a density exceeding the nuclear one $\rho_n \approx 2.6\times10^{14}$ g cm^{-3}. The possibility of phase transitions (PTs) in nuclear matter was first supposed by Gurevich [1]. Then Ivanenko & Kurdgelaidze [2] and Itoh [3] advanced hypotheses concerning stars composed of quark matter. Nowadays there exists an extensive literature on neutron stars, quark stars, and neutron stars containing quark cores (the so-called hybrid stars).

The properties of hybrid stars are of great importance for explaining the supernova explosion mechanism in the simplest case where there are no magnetic field and rotation. This is because the PT to quark matter that arises at the boundary between the core of a hybrid star and its crust can be responsible for the development of hydrodynamic instability ending with a supernova explosion.

Here we concentrate our attention on two unusual properties of hybrid stars: the existence of a “special point” on its mass-radius diagram and the “inverse” convection which can occur inside such a stars.

2. Special point

The published models of hybrid stars show a surprising peculiarity. On the mass–radius ($M-R$) diagram, all of the lines representing the sequences of models with different constant values of the bag constant B intersect in a very small region that we arbitrarily call a “point” here. To construct the stellar models, we use an equation of state (EOS) with the phase transition to quark matter at high densities. An approximation of the EOS from [4] is applied for the low density component of the matter. The quark component is described by the simplest version of the bag model in which the relation between pressure P and total energy per unit volume ε is linear:

$$P = \frac{1}{3}\left(\varepsilon - 4B\right), \qquad (1.1)$$

where B is the quark bag constant. This approximation is widely used in modeling the properties of quark matter and is a special case of the group of linear EOSs: $P = \alpha(\varepsilon - \varepsilon_0)$, where the dimensionless constant $0 \le \alpha \le 1$ means the square of the speed of sound measured in units of the speed of light, $\alpha = (c_s / c)^2$. The bag constant B is a free model parameter and, in the simplest case, is uniquely related to the density at which the phase transition begins.

The mass–radius diagram for hybrid stars calculated for our EOS is shown in Fig. 1.

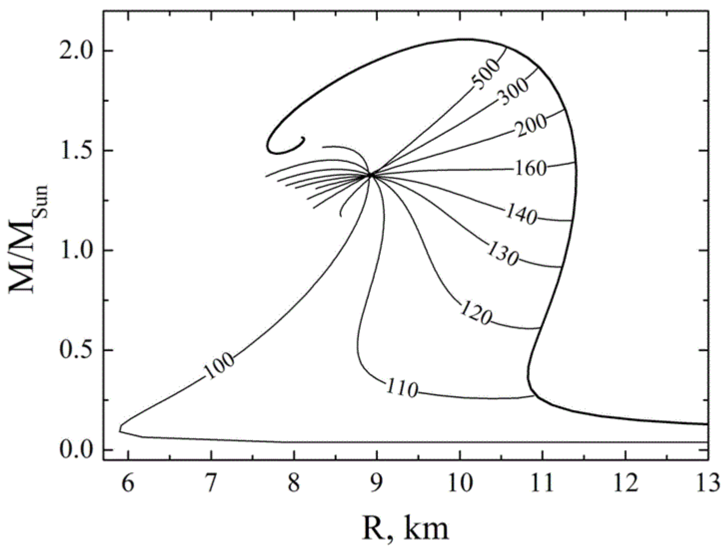

Fig1. *Mass–radius diagram of hybrid stars for various values of the parameter B.*

The thick line here indicates the dependence *M(R)* for the EOS without any phase transition to quark matter. The thin lines indicate these dependences for various values of the parameter B (the values of B are indicated by the numbers in units of $MeV\,fm^{-3}$). The density at which the phase transition begins ρ_1 is uniquely related to B. For example, $B = 120$ corresponds to $\rho_1 \approx 2\rho_n$, while for $B = 145$ we have $\rho_1 \approx 3\rho_n$. The curve with $B = 100$ describes an almost pure quark star with a thin crust made of ordinary matter and, therefore, exhibits a dependence *M(R)* typical of such stars. On the other hand, as can be seen from the figure, all stars with quark cores at $B \ge 160$ are unstable. Naturally, these specific values are unique to our model EOS.

Let us now turn to the formulation of the problem. As can be seen from Fig. 1, all curves with different B intersect in a very narrow region on the *(M−R)* diagram (but not at a point!). This property, which is surprising per se, not only leads to some interesting consequences that we will discuss in conclusion but also undoubtedly requires an explanation. Note also that such a behavior of the curves *M(R)* is not a unique property of precisely our EOS. The same effect can be seen, for example, in [5] – [7].

2.1. The brief explanation

Here we can only outline our explanation of this effect (for details see [8]). First, we need to compare the structures of stars near the point of intersection in Fig. 1. These stars corresponding to different values of the parameter B should have similar masses and radii. From this analysis, we see that these stars have a virtually identical crust made of ordinary matter to which a quark core is “stitched” at different depths, depending on the parameter B. Thus, when changing the parameter B, the quark matter–ordinary matter boundary is shifted, leaving the crust virtually unchanged.

We formalize this condition, starting from the stellar equilibrium equations under general relativity conditions (the Tolman–Oppenheimer–Volkoff equations) and the phase equilibrium conditions at the boundary. After some mathematical manipulations we obtained the main equation of the problem, which must hold for "special point" property existence. To solve this equation, we use the homologous variables trick and show that our quark EOS really satisfies this conditions to some extent (i.e. not exactly, that's why "a special point" in reality is a small domain).

The existence of "a special point" on the mass–radius diagram of hybrid stars appears to be a consequence of mostly two factors. The first one is the linearity of quark EOS (1.1). This property is characteristic of a simplest bag model of quark matter, but it also holds with very high accuracy for more realistic quark EOSs [9]. The second factor is a small value of the parameter $\alpha = 1/3$. The smaller is this parameter, the more exact is the homology property for the main equation. And inversely: at higher values of α the domain of intersection of *M(R)* curves would be broader.

2.2. Discussion

We established that the stars at "the special point" are "masked", hiding their true structure under the veil of observable quantities (*M* and *R*). Consider this aspect of the problem. Let us adopt the linearity of the quark EOS and assume that we know the true EOS of nuclear matter without any phase transitions that gives a thick enveloping curve on the ($M-R$) diagram (see Fig. 1). Then, were it not for the special point, only one measurement of the stellar mass and radius not only could say us whether such a star is a purely neutron or hybrid one (or, as a limiting case, a purely quark one) but could also point to parameters of quark matter. However, the existence of the special point changes the situation: the measuring of mass and radius of a star in its vicinity will only say us that this star contains a quark core, but neither its structure nor the parameters of quark matter will be determined. Either invoking additional information (for example, the cooling rate if the star was hot) or measuring the parameters of other hybrid stars to gain statistics and reconstruct the true curve *M*(*R*) will be required.

3. Inverse convection

In a hybrid star the quark core is separated from the outer nuclear matter envelope by an intermediate layer where the PT between nuclear and quark matter occurs. Within this region of coexistence of nuclear and quark phases there is a possibility that pressure decreases as temperature increases at constant density. Such an effect was mentioned, for instance, in [10] – [12]. In single phases of quark or hadronic matter usually the opposite is the case, i.e. the pressure increases with growing temperature at constant density. According to the Clapeyron–Clausius formula (see, e.g., [13]) the temperature derivative of pressure along the PT line in the phase diagram is given by:

$$\left(\frac{\partial P}{\partial T}\right)_{pt} = \frac{S_2 - S_1}{\frac{1}{\rho_2} - \frac{1}{\rho_1}} < 0. \qquad (1.2)$$

Here $S_{1,2}$ and $\rho_{1,2}$ are the entropies per baryon and densities of phases 1 and 2 in the region of their coexistence. We consider here the simplest Maxwellian description of PT. The negative sign of the derivative in equation (1.2) thus appears because the quark phase has a higher entropy per baryon, $S_2 > S_1$ (note that $\rho_2 > \rho_1$). This also implies that to go from the low-density phase 1 to the high-density phase 2 at a fixed temperature the system absorbs the thermal energy per baryon $\Delta q = T(S_2 - S_1) > 0$. As we will show in the next section, this property ensures very unusual convection properties in the layer of phase coexistence.

3.1. The conditions for convection

Let us consider a schematic view of the phase diagram in the density–temperature plane (Fig. 2):

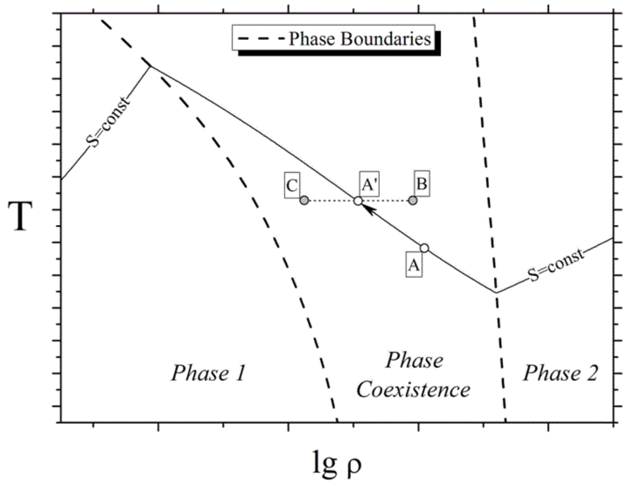

Fig2. *Schematic view of a phase diagram for the transition between hadronic and quark matter.*

The boundaries of the phases are shown by bold dashed lines. The region between the lines corresponds to two-phase coexistence. An isentropic curve (S = const) is shown by the solid line. One can observe that in the phase coexistence region the temperature begins to decrease along the isentropic curve when the density increases. Such a behavior of the PT was indicated by a number of authors (see e.g. [14]).

Consider for example a convective element that starts its adiabatic motion in the interior of a hybrid star at point A and moves outwards reaching point A'. So it keeps the initial entropy but has along its way the same pressure and hence temperature (let us remind that we consider the Maxwellian PT) as the environment. If on its way the convective element proves to be denser than the environment (point C) it will begin to sink. Such a configuration is convectively stable. On the contrary, when the state of the environment corresponds to point B, the density of the convective element is lower than that of the environment and it continues to rise – the configuration is convectively unstable. The entropy of environment in point B is higher than in point A'. Therefore the condition of appearance of convection reads

$$\frac{dS}{dr} > 0. \tag{1.3}$$

This condition has the inequality sign opposite to the common Schwarzschild criterion.

We can write the general criterion of appearance of convection in the well-known Ledoux form:

$$\left(\frac{\partial \varepsilon}{\partial S}\right)_{P,Y} \frac{dS}{dr} + \left(\frac{\partial \varepsilon}{\partial Y}\right)_{P,S} \frac{dY}{dr} > 0. \tag{1.4}$$

For simplicity we assume below Y = const (chemically uniform conditions). Then the onset of convection depends on the distribution of entropy in the star (the term dS/dr in equation (1.4)) and the sign of the term $\left(\frac{\partial \varepsilon}{\partial S}\right)_{P,Y}$. In the nonrelativistic limit we have $\varepsilon \approx \rho c^2$ with c being the baryon mass density. Therefore up to a factor c^2 the multiplier in front of dS/dr is given by

$$\left(\frac{\partial \rho}{\partial S}\right)_P = -\frac{\rho^2}{\left(\frac{\partial P}{\partial T}\right)_S} = -\frac{T\rho}{P\gamma c_V}\left(\frac{\partial P}{\partial T}\right)_\rho, \tag{1.5}$$

where we introduce the adiabatic index γ and the specific heat capacity c_V. For matter under common conditions the right-hand side of equation (1.5) is obviously strictly negative and we obtain the criterion for onset of convection in the Schwarzschild form:

$$\frac{dS}{dr} < 0. \tag{1.6}$$

Hence, a negative gradient of entropy causes the onset of convection. However, within the phase-coexistence region the derivatives $\left(\frac{\partial P}{\partial T}\right)_S$ and $\left(\frac{\partial P}{\partial T}\right)_\rho$ can be negative and as a result the Schwarzschild criterion changes its sign. The negative entropy gradient in this region ensures the convective stability while the positive one stimulates the development of a convective instability. Let us consider now the general case. One can easily show that the factor in front of d*S/*d*r* in equation (1.4) is equal to

$$\left(\frac{\partial \varepsilon}{\partial S}\right)_{P,Y} = \rho T\left[1 - \frac{\varepsilon + P}{T\left(\frac{\partial P}{\partial T}\right)_S}\right]. \tag{1.7}$$

In the non-relativistic limit the second term in square brackets is much larger than 1 and equivalent to equation (1.5). If $\left(\frac{\partial P}{\partial T}\right)_S < 0$, the sign of this term as well as the criterion of convection change again.

3.2. Discussion and conclusions

We need to emphasize that the possibility of “inverse” convection discussed here is the direct consequence of the unusual property of the deconfinement PT expressed by equation (1.2). For example, for the nuclear liquid–gas PT the condition for convection will have the ordinary form as for single phases of quark and hadronic matter.

The consequences of possible existence of the “inverse” convection zone in a hybrid star can be conceived by looking at Fig.2. In the case of well-developed convection an ascending matter element that travels from *A* to *A*’ has lower entropy than the environment and thus lower heat content. Similarly, a descending element has higher heat content than the environment. Therefore in contrast to ordinary convection, within the “inverse” convection zone the heat flux is directed inwards in a star.

Currently, the neutrino-driven scenario is the favored explosion mechanism of core-collapse supernovae. In this scenario turbulence and convection are crucial to achieve sufficient neutrino heating of matter to trigger an explosion. If the “inverse” convection appeared already in the early post-bounce phase of the supernovae, in principle it could also impact the explosion dynamics. These possibilities require further investigation by detailed numerical simulations.

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