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The magnetic field origin provided by vortex-inflowing processes in sunspots

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Abstract. As is well known, the processes of vortex-inflowing occur in sunspots, i.e. there exist circular motion of solar matter along with its simultaneous inflowing to the central region. Magnetic fields are formed at the depths of several ths kilometers, where the plasma temperature amounts to 100 000 K and more. In these circumstances plasma is totally ionized. Since solar plasma by 90% consists of hydrogen, it may be considered as a mixture of the proton fluid and the electron fluid. Kinematical viscosities of proton and electron fluids relates to each other in inversely proportion to square roots of proton and electron masses, i.e. the electron fluid is 42,8 times more viscous than proton fluid. As a result the electron fluid in its vortex-inflowing motion to the center of a sunspot is slowed down because of friction with the electron fluid situated far from the center of the inflow region and lag in its circular motion in comparison with the proton fluid. The larger velocity of proton liquid means the presence of circular currents and hence the generation of magnetic field. In the context of the present article the efficiency of the mechanism of magnetic field formation from a "zero"-state with reference to sunspots is demonstrated; preliminary calculation of the sunspot magnetic field in the context of proposed model is performed and a comparison with existing data is made, which shows good agreement with the existing observational data and up-to-date conceptions.

Key words: Sunspot magnetic induction – vortex-inflow – rate of inflow – plane stationary vortical process – proton (electron) viscosity

The Sun ranks among common stars: its general magnetic field is characterized by induction of no more than 1–2 G. But magnetic fields of intensive sunspots usually amount to 1000–2000 G and more. The observed sunspot diameter is usually of the order of 10000 km.

Sunspots have two regions — umbra and penumbra. The sunspot umbra is situated in the central part of a sunspot; it is darker than the penumbra and has an almost uniform structure. The umbra diameter approximately amounts to a half of the whole sunspot diameter. Penumbra gathers round the umbra like a ring; in contrast to the umbra, the penumbra has a pronounced non-uniform structure, which consists of a great number of fibrils and granules.

It is known that the Evershed effect is observed in sunspots — well-ordered gas motion in radial, tangential and vertical directions. The maximum of radial and tangential velocities fall at the boundary between the umbra and penumbra (Gopasyuk, 1976). At the visible surface of the photosphere the outward gas motion from a sunspot exceeds the inward motion. This allows one to conclude, relying on the law of conservation of matter, that at the sunspot formation depth the inflow exceeds the outflow. Then, the vortical structure of the penumbra fibrils is clearly observed on many sunspot photos (Fig. 1) (Molodensky, Filippov, 1992). It means that a vortex-inflow occurs

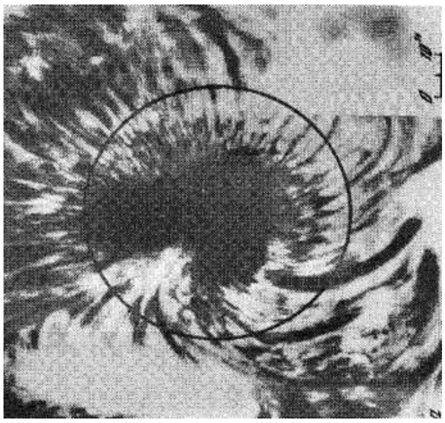

Figure 1: Vortex-inflow in a sunspot.

in sunspots, i.e. the circulation of solar substance takes place along with its simultaneous flow to the central region. In this respect, sunspots are similar to mighty Earth's cyclones; the sunspot umbra is similar to the eye of hurricane of a cyclone.

For a sunspot we originally admitted a simplified plane stationary hydrodynamic model in the form of a viscous liquid vortex-inflow (Fig. 2) (Loytsyansky, 2003). The motion of such liquid is described by Navier–Stokes equation. We use one of its forms:

$$\frac{\partial \Omega}{\partial t} + (\mathbf{V}\nabla)\Omega = \nu \Delta \Omega \,, \qquad (1)$$

where $\mathbf{V}$ is the velocity of plasma motion, ν is the kinematical plasma viscosity, $\Omega = \operatorname{curl} \mathbf{V}$ is vorticity.

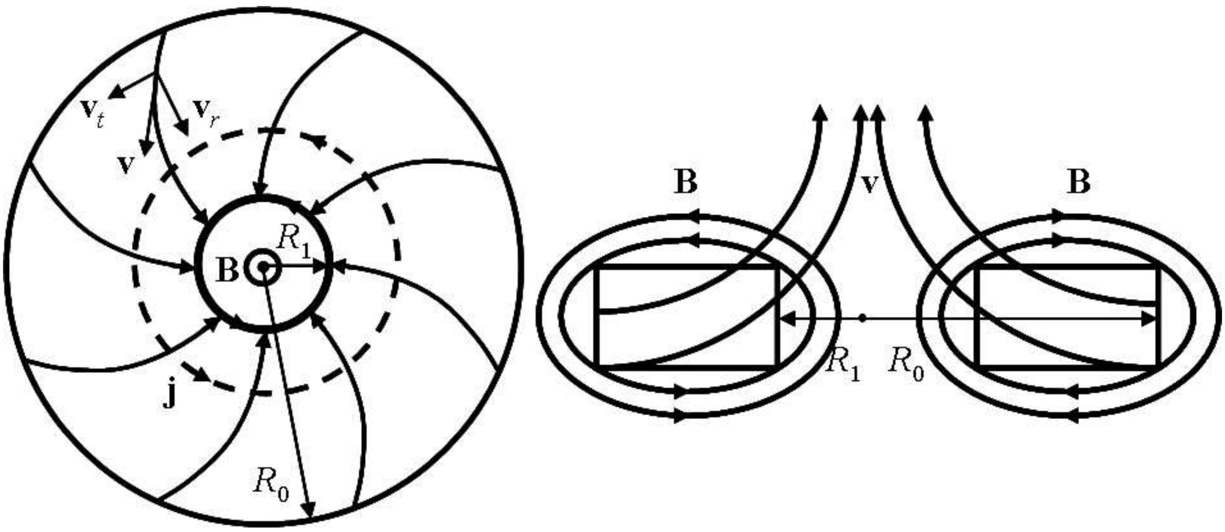

Figure 2: Magnetic field at vortex processes in plasma. On the left is a plane model of vortex-inflow, on the right is the vertical cross section.

The vortex-inflow has central symmetry, therefore the point of its origin we superpose with the center of a circle of radius R_1, the circle is the inner boundary of the fluid region. Owing to the central symmetry, radial and tangential components of plasma velocity and also vorticity depends only on the radius r: $v_r = v_r(r)$, $v_t = v_t(r)$, $\Omega_z = \Omega_z(r)$. The time derivative of vorticity is equal to zero: $\partial\Omega/\partial t = 0$ because of stationarity of the process.

The vortex-inflow as a whole is characterized by the rate of inflow Q, and Q is negative in the case of inflow. In accordance with the continuity equation, the rate of inflow remains the same in any point of the inflow region: $Q = 2\pi r v_r = const < 0$. Solving Navier–Stokes equation (1), we obtain the following expressions for vorticity (here and further the subscripts "z" are dropped):

$$\Omega(r) = \frac{2v_{t1}}{R_1}\left(\frac{r}{R_1}\right)^k, \tag{2}$$

where v_{t1} is the magnitude of tangential velocity at the inflow region boundary, k is the parameter of the inflow:

$$k = \frac{Q}{2\pi\nu} \tag{3}$$

which characterizes the ratio of the rate of inflow to kinematical viscosity ν. Since $Q < 0$, k is negative too; in fact a stronger condition is required: $k < -2$ (Kaplan, Otkidychev, 2006).

Let us perform specific calculation of the plasma velocity at the depth of sunspot formation and estimate maximum values of v_t and v_r. Magnetic fields originate at depths of several thousand kilometers, where the plasma temperature is of the order of 10^5 K. To estimate the maximum plasma motion velocity, we use the Bernoulli theorem:

$$p + \rho v^2/2 = p_0, \tag{4}$$

where p and p_0 are the magnitudes of pressure inside and outside the vortical process, respectively. The ratio of p_0 to p we can find from the equation of adiabat:

$$\left(\frac{p}{p_0}\right)^{n-1}\left(\frac{T}{T_0}\right)^n = const, \tag{5}$$

where n is the adiabatic index, which is equal to $5/3$ for completely ionized plasma. The temperature of a sunspot at the photosphere surface is approximately $b = 1.5$ times lower than the temperature of a quiet Sun at the photosphere surface. We consider that the ratio of temperatures inside and outside the sunspot at the depth of several thousand kilometers underneath the photosphere is considerably less and equals $b \equiv 1 + \alpha = 1.1$ $(\alpha = 0.1)$. Hence, the decrease of temperature by a factor of 1.1 in accordance with equation of adiabat results to the decrease of pressure in $b^{5/2} \approx 1.25$ times, i.e.

$$p = b^{-5/2}p_0 \approx 0.75p_0, \quad v^2 = 2(p - p_0)/\rho = 0.5p_0/\rho. \tag{6}$$

Taking into consideration the Mendeleyev–Clapeyron equation, the expression for the plasma velocity squared is represented as follows:

$$v^2 = 5\alpha\frac{p_0}{\rho_0} = 5\alpha\frac{RT}{\mu} = \frac{5\alpha \cdot 8.31 \cdot T}{0.5 \cdot 10^{-3}} = 8.31 \cdot 10^4 \alpha T\,\frac{\mathrm{m}^2}{\mathrm{s}^2}, \tag{7}$$

where we have taken into account that molar mass μ of ionized hydrogen plasma is half as large as molar mass of monatomic hydrogen. The velocity corresponding to a temperature of 10^5 K is $v = \sqrt{8.31 \cdot 10^4 \cdot 0.1 \cdot 10^5\,\mathrm{m/s}} = 28\,\mathrm{km/s}$. At the surface of a sunspot radial and tangential

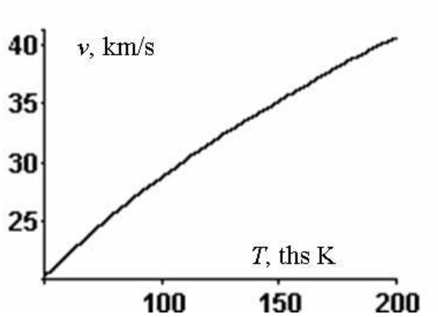

Figure 3: Relation between plasma velocity and its temperature.

components have the same order, although v_r is slightly higher than v_t. We consider that at the depth of formation of a sunspot $v_r^2 \approx v_t^2 = v^2/2$, and $v_r \approx v_t = 20\,\mathrm{km/s}$. The graph of plasma velocity depending on temperature is represented in Fig. 3.

Formula (2) shows that for the fixed value of the rate of inflow the greater is the decrease in k, the more rapidly Q decreases with distance. The schematic images of the vortex-inflowing of two liquids are represented in Fig. 4: the less viscous liquid is shown on the left side of the picture and the more viscous liquid is on the right side.

Now let us consider the vortex-inflowing plasma motion in a sunspot. Magnetic fields form at depths of tens of thousands of kilometers; in accordance with present opinions the depth of the formation zone increases with increasing magnitude and magnetic field of a sunspot. Farther we mean that the temperature range at which the magnetic field forms is 50000–200000 K, the characteristic temperature is 100000 K.

Under such conditions plasma is totally ionized. Since hydrogen accounts for 90% (by the number of atoms) of the composition of the Sun, the solar plasma can be considered to be a mixture of two fluids: the proton fluid and the electron fluid. Dynamic viscosities of proton and electron fluids relates to each other proportional to square roots of proton and electron masses (Lifshits, Pitayevsky, 2002).

$$\frac{\eta_i}{\eta_e} = \sqrt{\frac{m_i}{m_e}} \tag{8}$$

Hence, kinematical viscosities of proton and electron fluids relate to each other inversely proportional to square roots of proton and electron masses:

$$\frac{\nu_i}{\nu_e} = \sqrt{\frac{m_e}{m_i}} \tag{9}$$

Thus the electron fluid is 42.8 times more viscous than that of protons. This means that the properties of the electron fluid are determined by its viscosity, while the properties of the proton fluid are determined by its inertia. As a result, the electron fluid in its vortex-inflowing motion to the center of a sunspot is slowed as caused by friction with the electron fluid situated far from the center of inflow region and lag in its circular motion in comparison with that of protons. The larger velocity of proton liquid implies the presence of circular currents and hence generation of magnetic field.

If proton and electron viscosities could exist by themselves, the left picture in Fig. 4 must correspond to the vortex-inflowing motion of the proton liquid and the right one to the electron liquid. However, in fact, proton and electron liquids could not be separated from each other, therefore their mutual motion would be such as it is represented in Fig. 5.

Using (2), the basic calculation formula is obtained in (Kaplan, Otkidychev, 2006). This formula determines the relationship between magnetic induction B_1 and vorticity Ω_1 at the vortex-inflow region boundary R_1:

$$B_1 = \frac{a\mu_0\gamma Q\Omega_1}{4\pi(k+2)} \tag{10}$$

where $a = 2.436 \cdot 10^{-10}\,\mathrm{kg/C}$ is the numerical coefficient, m_i and m_e are the masses of proton and electron, respectively, x is the elementary charge, μ_0 is the magnetic constant: $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{H/m}$.

Formula (10) has a clear physical sense.

1. The modulus of the rate of inflow must be large enough for the inflow parameter k to be less than the threshold magnitude. As it was mentioned above, the magnitude of the rate of inflow is negative because it is considered as the minus-source.

2. Magnetic induction in the central region of the vortical process is proportional to the plasma conductivity, the rate of inflow and plasma vorticity in the inner region of vortical process.

In (Kaplan, Otkidychev, 2006) formula (10) is presented approximately in the following form:

$$B_{R_1} = a\mu_0\gamma v^2/2. \tag{11}$$

Here the unit is tesla:

$$|B| = |a| \cdot |\mu_0| \cdot |\gamma| \cdot |v|^2 = \frac{\mathrm{kg}}{\mathrm{C}}\frac{\mathrm{H}}{\mathrm{m}}\frac{\mathrm{mho}}{\mathrm{m}}\frac{\mathrm{m}^2}{\mathrm{s}^2} = \mathrm{tesla} = 10^4\ \mathrm{G}.$$

Let us find the magnetic induction dependence on temperature T. The speed dependence on plasma temperature was obtained above in formula (7). We have taken the formula for the plasma conductivity from (Priest, 1985):

$$\gamma = 1.53 \cdot 10^{-2} T^{3/2}/L_e\,\mathrm{mho/m}, \tag{12}$$

where L_e is Coulomb logarithm; its value for upper layers of the photosphere calculated by the formula from (Priest, Forbes, 2002) is 3.5. Hence,

$$\gamma = 4.37 \cdot 10^{-3} T^{3/2}\,\mathrm{mho/m}. \tag{13}$$

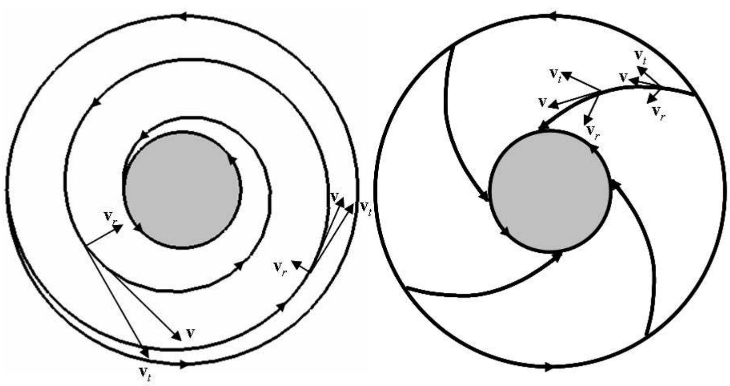

Figure 4: The schematic images of the stationary vortex-inflow motion of the less viscous liquid (on the left) and the more viscous liquid (on the right) without allowing for mutual friction.

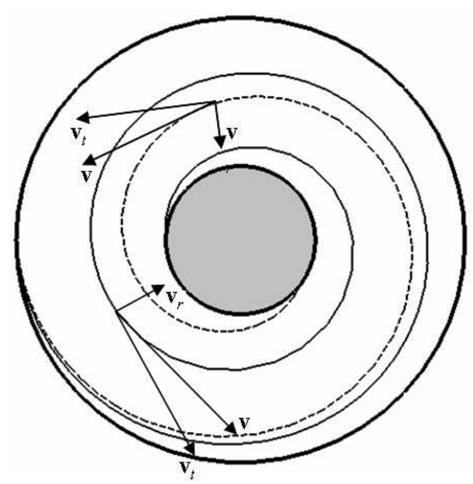

Figure 5: The schematic image of the real motion of the proton liquid (continuous line) and electron liquid (dotted line) at stationary vortex-inflow.

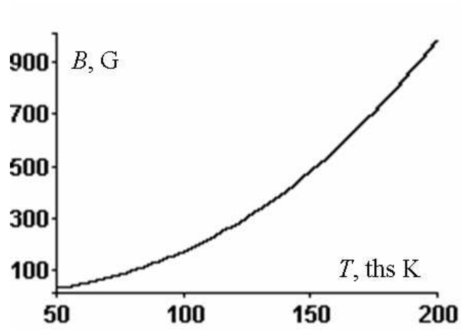

Figure 6: The dependence of magnetic induction on temperature.

Substituting the dependences of velocity and conductivity on temperature in the formula for the magnetic induction (11), we obtain the magnetic induction dependence on temperature at the inner boundary of the inflow region:

$$B_{R_1}(T) = 2.44 \cdot 10^{-10}\frac{\mathrm{kg}}{\mathrm{C}} \cdot 4 \cdot 3.14 \cdot 10^{-7}\frac{\mathrm{H}}{\mathrm{m}} \cdot 8.31 \cdot 10^{4}\alpha T\,\frac{\mathrm{m}}{\mathrm{s}} \cdot 4.37 \cdot 10^{-3}\frac{\mathrm{mho}}{\mathrm{m}},$$

$$B_{R_1}(T) = 5.55 \cdot 10^{-14}\alpha T^{5/2}\,\mathrm{tesla} = 5.55 \cdot 10^{-10}\alpha T^{5/2}\,\mathrm{G}. \tag{14}$$

Thus the magnetic induction is proportional to temperature in the power of $5/2$.

In accordance with up-to-date conceptions, depths of the sunspot magnetic field formation amount to 20000 km (Jahn, 1991). The temperature gradient of upper layers of the convection zone is about 10 kelvin/km. Such a depth corresponds to a temperature of the order of 200000 K. The induction dependence on temperature is shown in Fig. 6 in a range of 50000–200000 K. In such a temperature range and when $\alpha = 0.1$, the magnetic induction varies in a range of 31–990 G, at that temperature 100000 K corresponds to induction 175 G.

Thus, in the context of the present article the following conclusions were drawn:

- The efficiency of proposed earlier mechanism of magnetic field formation from a "zero state" with reference to sunspots is demonstrated;
- The reductive hydrodynamic plane sunspot model is suggested, which gives a simple explanation of a visible picture of a sunspot;
- The preliminary calculation of sunspot parameters in the context of the proposed model is realized and a comparison with the existing data is made.

The performed comparison shows that the suggested simplified model of the sun-spot structure and the mechanism of magnetic field formation correspond well enough to the available observational data and prevalent conceptions.

It is clear that in the context of simplified flat model it is impossible to describe sunspot completely. A set of questions remains a problem, in particular:

- There are not enough data in order to confidently talk about the temperature at the depth of sunspot formation;
- The magnetic field at the level of formation is determined in this paper, but nothing is said about its spatial structure and about the mechanism of magnetic field rise to the surface;
- The theory is applied to a single sunspot, while sunspots almost always appear in groups.

To remove these shortcomings, one has to build a three-dimensional magneto-hydrodynamic model and, probably, draw additional theoretical approaches, particularly a dynamo-mechanism.

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